Test ID:- C21

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SOLUTIONS

CIVIL ENGINEERING : PAPER-1

Full Length Test -15

Exam Date :- 01 March 2026

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2 Marks

Q.1

Sol. **Neutral Axis and Moment of Resistance**

- **Neutral Axis (N.A.):**
  It is the axis in the beam's cross-section where **bending stress is zero**. Fibres along the N.A. do not change in length.
- **Moment of Resistance (MR):**
  The internal resisting moment developed by the stresses in a cross-section, equal to the applied bending moment.

  $$MR = \sigma_{max.} Z$$

  where $Z = I / y_{max}$ is the section modulus.

Q.2

Sol. Magnification factor $(R_d) = \dfrac{1}{1-\beta^2}$

where, $\beta = \dfrac{w}{w_n}$

& w - excitation frequency
w_n - Natural frequency of system

Q.3

Sol. Distribution factor for a member at a joint is the ratio of the stiffness (or relative stiffness) of the member to the total stiffness (or total relative stiffness) of all the members meeting at that joint.

$$DF = \frac{\text{Stiffness of the member}}{\text{Total stiffness of that joint}}$$

or

$$DF = \frac{\text{Relative stiffness of the member}}{\text{Total stiffness of the joint}}$$

Q.4

Sol.
- To determine the maximum value of reaction, shear force, or bending moment at any section of a structure due to moving loads.
- To find the position of moving load which produces the maximum effect (reaction, shear, or moment) in a structure.

Q.5

Sol. The degree of freedom (D.O.F.) of a structure is the number of independent displacements or rotations required to define its configuration or movement.

In a 2D structure, each joint may have up to three degrees of freedom (two translations and one rotation).

A structure is kinematically determinate when all degrees of freedom are restrained; otherwise, it is kinematically indeterminate.

Q.6

Sol. 1.5 DL + 1.5 L.L = (1.5 × 25) + (1.5 × 35) = 90 kN-m

1.2(DL + L.L + S.L) = 1.2(25 + 35 + 15) = 90 kN-m

So, Design B.M. = 90 kN-m

Q.7

Sol. It is stretched element in a concrete member to pre-stress to the concrete, in general, high tensile steel wires are used as tendos.

Q.8

Sol.

$$S.R. = \frac{l}{r}$$

r - radius of gyration $\sqrt{\frac{I}{A}}$

$$= \sqrt{\frac{7.85\times10^{-5}}{\frac{\pi}{4}\times(0.2)^2}} = 0.049$$

$$\text{Slenderness Ratio} = \frac{4.9}{0.049} = 85.71$$

Q.9

Sol. Radius of gyration of a body is defined as the distance between the axis of rotation and a point at which the whole mass of the body is supposed to be concentrated, so as to possess the same moment of inertia as that of body.

Q.10

Sol. The secant formula is also based on the same assumptions as the Euler's formula. However's in the secant formula, load is not axial, but has small eccentricity e.

Q.11

Sol. The stress distribution in longitudinal fillet welds in shown in Fig. The stress distribution is non-uniform with higher stresses at the two ends. Because of this, the stress concentration occurs in due course of time and the failure of weld initiates from here. When end returns of two times the size of weld are provided the failure is checked.

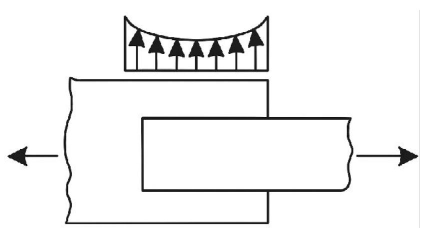


Fig. Stress pattern in fillet weld without end returns

Q.12

Sol. It is defined as the ratio of the moment at the fixed far end to the moment at the rotating near end.

$$\text{C.O.F.} = \frac{\text{Carry over moment at farther end}}{\text{applied moment at near end}}$$

Q.13

Sol. A beam in which compression flange is not restrained in lateral direction to prevent buckling or crippling in the steel beam, is known laterally unsupported or unrestrained beam.

Q.14

Sol. Void ratio, $e = 0.7$

Specific gravity, $G_s = 2.67$

Critical hydraulic gradient

$$i_{cr} = \frac{G_s - 1}{1+e} = \frac{2.67-1}{1+0.7} = 0.98$$

Q.15

Sol. Given, H = 3.6m, $\phi = 30^o$, $\rho = 1.9\,gm/cc$

Active earth Pressure at a depth of 3.6m $(\rho_a) = k_a.\gamma.H$

$$= \left(\frac{1-\sin 30^o}{1+\sin 30^o}\right) \times 1.9 \times 9.81 \times 3.6$$

$$= 22.37\ kN/m^2$$

Q.16

Sol. **(i) Both ends fixed**

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

(ii) One end fixed and the other free

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

Additional:

(iii) Both ends hinged

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

(iv) One end fixed and the other hinged

$$P_{cr} = \frac{2\pi^2 EI}{L^2}$$

Q.17

Sol. Bearing capacity of footing on purely cohesive soil does not depend upon size of footing.

$$q_{up} = q_{uf} \text{(for clay)}$$

$\therefore$ Ultimate bearing capacity of a 2m wide square footing = 180 kPa

Q.18

Sol. Auger boring, also known as horizontal earth boring or jack and bore, is a trenchless construction method used to install pipes and conduits underground with minimal surface disruption. This technique is ideal for crossing beneath obstacles like roads, railways, and existing structures without open-cut excavation.

Q.19

Sol. Main types of damping:

(1) Structural damping

(2) Viscous damping

(3) Coulomb damping

We generally idealise damping in structure as viscous damping.

Q.20

Sol. Relation between Bending Moment and Bending Stress

Bending moment is directly proportional to bending stress:

$$M = \frac{\sigma_{max} \times I}{y} = \sigma_{max}.Z$$

Thus,

$$\sigma = \frac{M.y}{I}$$

Greater the bending moment, higher will be the bending stress in the beam.

5 Marks

Q.21

Sol. Given : Decrease in length (δl) = 0.25 mm; Modulus of elasticity for steel (E_S) = 210 GPa = 210 × 10^3 N/mm^2; Modulus of elasticity for aluminium (E_A) = 70 GPa = 70 × 10^3 N/mm^2; Area of steel section A_S = 50 × 50 = 2500 mm^2; Area of aluminium section (A_A) = 100 × 100 = 10000 mm^2; Length of steel section (l_S) = 300 mm and length of aluminium section (l_A) = 380 mm.

Let P = Magnitude of the force in kN.

We know that decrease in the length of the member (δl).

$$0.25 = P\left(\frac{l_S}{A_S E_S} + \frac{l_A}{A_A E_A}\right)$$

$$0.25 = P\left(\frac{300}{2500 \times (210 \times 10^3)} + \frac{380}{10000 \times (70 \times 10^3)}\right)$$

$$0.25 = \frac{780P}{700 \times 10^6}$$

$$P = \frac{0.25 \times (700 \times 10^6)}{780} = 224.4 \times 10^3 \text{ N} = 224.4 \text{ kN}$$

Q.22

Sol. Given : Length (l) = 2m = 2 × 10^3 mm ; Width (b) = 20 mm ; Thickness (t) = 15 mm ; Tensile load (P) = 30 kN = 30 × 10^3 N ; Poisson's ratio $\left(\frac{1}{m}\right)$ = 0.25 or m = 4 and Young's modulus of elasticity (E) = 200 GPa = 200 × 10^3 N/mm^2.

Let δV = Increase in volume of the bar.

We know that original volume of the bar.

$$V = l.b.t = (2 \times 10^3) \times 20 \times 15 = 600 \times 10^3 \text{ mm}^3$$

and

$$\frac{\delta V}{V} = \frac{P}{btE}\left(1 - \frac{2}{m}\right) = \frac{30 \times 10^3}{20 \times 15 \times (200 \times 10^3)}\left(1 - \frac{2}{4}\right) = 0.00025$$

$$\delta V = 0.00025 \times V = 0.00025 \times (600 \times 10^3) = 150 \text{ mm}^3$$

Q.23

Sol. **Given :** Diameter of the shaft (D) = 100 mm ; Power transmitted (P) = 120 kW and speed of the shaft (N) = 150 r.p.m.

Let T = Torque transmitted by the shaft, and

τ = Intensity of shear stress in the shaft.

We know that power transmitted by the shaft (P).

$$120 = \frac{2\pi NT}{60} = \frac{2\pi \times 150 \times T}{60} = 15.7\,T$$

$$\therefore \quad T = \frac{120}{15.7} = 7.64\,kN-m = 7.64\times10^{6}\,N-mm$$

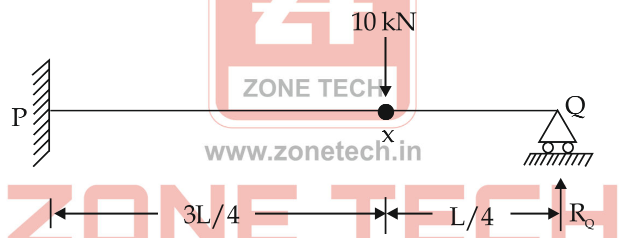
We also know that torque transmitted by the shaft (T).

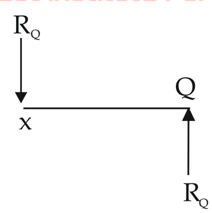
$$7.64\times10^{6} = \frac{\pi}{16}\times\tau\times D^{3} = \frac{\pi}{16}\times\tau\times(100)^{3} = 0.196\times10^{6}\,\tau$$

$$\tau = \frac{7.64}{0.196} = 39\,N/mm^{2} = 39\,MPa$$

Q.24

Sol.





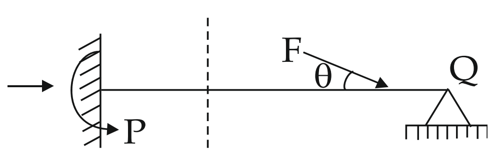
Taking right side of hinge

$$\sum mx = 0\,(\text{Bending moment at a hinge})$$

$$R_Q \times \frac{L}{4} = 0 \Rightarrow \boxed{R_Q = 0}$$

Q.25

Sol.



Ds = 3C – R
= 3 × 1-1
= 2

Dk = 3J - r
= 3 × 2 - 5
= 1

Q.26

Sol. For an isolated T-beam,

For simply supported beam, $L_0 = L_{eff}$

Effective width of flange,

$$b_f = \frac{l_0}{\left(\frac{l_0}{B}+4\right)} + b_w = \frac{6\times 1000}{\frac{6000}{1000}+4} + 300 = 900\,\text{mm}$$

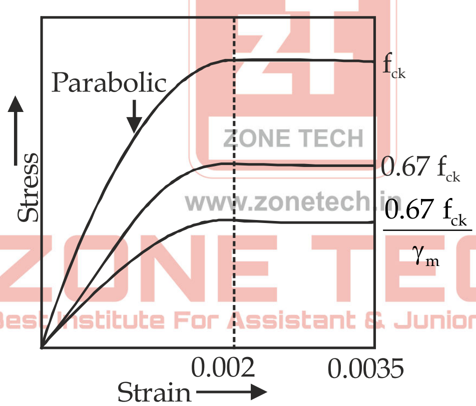
Q.27

Sol. **As per IS 456: 2000 Clause (39.1)** design of the limit state of collapse in compression shall be based on following assumptions:

(a) Plane section normal to the axis remain plane after bending.

(b) The maximum compressive strain in concrete in axial compression is taken as 0.002.

(c) The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.



Strain Curve for concrete

(d) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the result of test. An acceptable stress-strain curve is given in figure. For design purpose, the compressive strength of concrete in structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor γ_m = 1.5 shall be applied in addition to this.

Q.28

Sol. Loss of prestress due to anchorage slip

$$= \frac{\Delta L}{L}\times E_s = \frac{3\times 10^{-3}}{30}\times 2.1\times 10^5$$

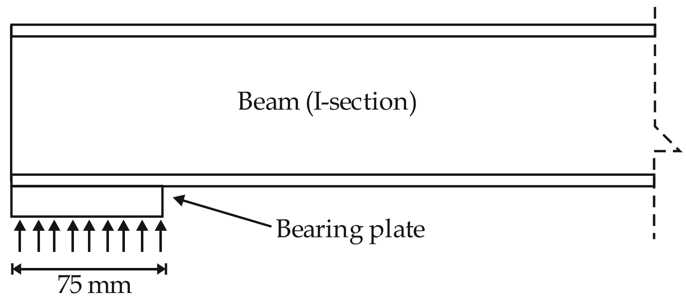
$$= 21\ \text{N/mm}^2$$

% loss of prestress

$$= \frac{21}{1200}\times 100 = 1.75\%$$

Q.29

Sol.



$$\text{Flange width (beam)} = 20 \text{ mm}$$
$$\text{Web width (beam)} = 8\text{mm}$$
$$F_y = 250 \text{ MPa}$$
$$\gamma_{mo} = 1.10$$
$$\text{Root radius of I-section} = 10\text{mm}$$

$$\text{Web bearing strength} = \left[b + 2.5(t_f + R)\right] t_w \frac{f_y}{\gamma_{mo}}$$

$$= \left[75 + 2.5(20 + 10)\right] \times 8 \times \frac{250}{1.1} = 272.72 \text{ kN}$$

$$= \boxed{272.73\,\text{kN}}$$

Q.30

Sol. Tension Member Subjected to Axial Load

The following procedure may be followed in the design of an axially loaded tension member.

1. The net area required (A_{net}) to carry the design load P is obtained by the equation,

   $$P = \sigma_{at} A_{net}$$

2. The net area calculated thus, is increased suitably (25%–40%) to compute the gross sectional area. From I.S. Handbook No. 1 suitable section/sections providing a cross-sectional area matching with the computed gross sectional area is selected.
3. The number of rivets required to make the connection is calculated. These are arranged in a suitable pattern and the net area of the section provided is calculated. This should be more than the net area calculated in step (1).
4. The slenderness ratio of the member is checked as per the I.S. specifications.

Q.31

Sol. Initial pressure at the centre of the clay layer.

$$\bar{\sigma}_0 = 4 \times 20 + 1.25 \times 10 = 102.5 \text{ kN/m}^2$$

From equation 12.58,

$$s_f = \frac{C_e}{1 + e_0} . H_0 \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$$

$$= \frac{0.22}{1 + 1.30} \times 250 \log_{10}\left(\frac{102.5 + 30.0}{102.5}\right)$$

$$= 0.0263\,\text{m} = 2.63\,\text{m}$$

When the water table rises to the ground surface,

$$\bar{\sigma}_0 = 4\times(20-10)+1.25\times(18-10) = 50\ \text{kN/m}^2$$

Therefore, $$s_f = \frac{0.22}{1+1.30}\times 2.50\log_{10}\left(\frac{50+30}{50}\right)$$

$$= 0.0488\,\text{m} = 4.88\,\text{cm}$$

As expected, the settlement increases due to the rise of the water table.

Q.32

Sol. Stiffness of each spring (k) $= \frac{F}{\Delta}$

$$\Rightarrow \quad k = \frac{300}{5} = 60\ \text{N/mm} = 60\ \text{kN/m}$$

Equivalent stiffness of system (k_{eq}) = 4 k

$$\Rightarrow \quad k_{eq} = 240\ \text{kN/m}$$

Now,The resonance will occur at natural frequency of system

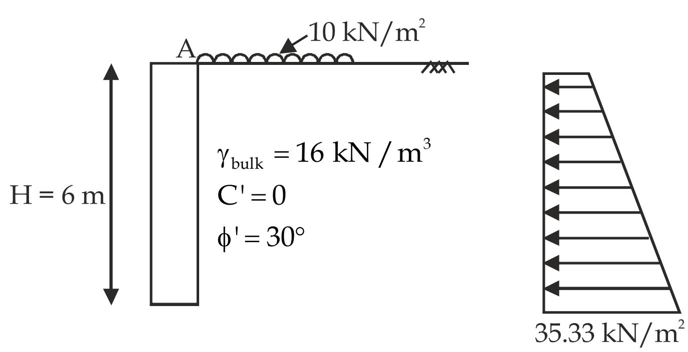
$$w_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{240\times10^3}{300}}$$

$$w_n = 28.28\ \text{rad/s} = 270.05\ \text{rpm}$$

20 Marks

Q.33

Sol. (i)



Active earth pressure diagram

Coefficient of active earth pressure, $K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$

Active earth pressure at top of wall $= K_a q = \frac{1}{3}\times 10 = 3.33\ \text{kN/m}^2$

Active earth pressure at bottom of wall $= K_a\gamma_{bulk}H + K_a q$

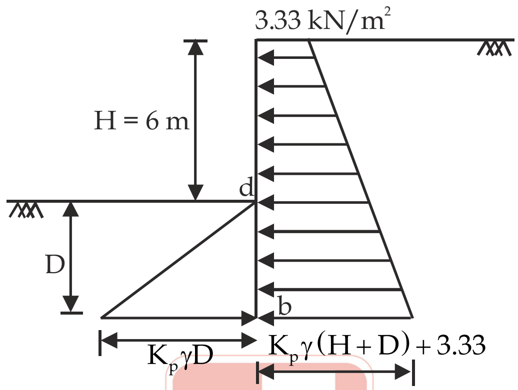
$$= \frac{1}{3}\times16\times6+3.33 = 35.33\ \text{kN/m}^2$$

Active thrust= Area of pressure diagram

$$= (3.33 \times 6) + \frac{1}{2} \times 6 \times (35.33 - 3.33)$$

= 116 kN/m – length of wall

(ii) Let the depth of embedment of cantilever sheet pile be D m.
Using approximate analysis



Taking moment of all forces about b

$$\frac{1}{2}K_p\gamma D^2 \times \frac{D}{3} = 3.33(H+D)\times\frac{(H+D)}{2} + \frac{1}{2}K_a\gamma(H+D)^2 \times \left(\frac{H+D}{3}\right)$$

Coefficient of passive earth pressure, $K_p = \frac{1}{K_a} = 3$

$$\therefore \quad \frac{1}{2}\times 3 \times 16\frac{D^3}{3} = \frac{3.33}{2}(6+D)^2 + \frac{1}{2}\times\frac{1}{3}\times 16 \times \frac{(6+D)^3}{3}$$

$$8D^3 = 1.665(36 + D^2 + 12D) + 0.889(216 + D^3 + 180^2 + 108D)$$

$$8D^3 = 59.94 + 1.665D^2 + 20D + 192 + 0.889D^3 + 16D^2 + 96D$$

$$7.11D^3 - 17.665D^2 - 116D - 251.94 = 0$$

D = 6.1 m

Since it is an approximate method, hence allowing 20% extra,

Depth of embankment = 1.2 × 6.1

= 7.32 m

Q.34

Sol. $\Sigma F_V = 0$

$R_A + R_B = 10 + 16$

$\Sigma M_A = 0$

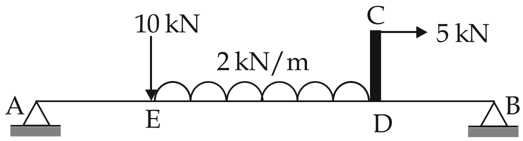
$24R_B - 2.5 - (10 \times 8) - (2 \times 8 \times 12) = 0$

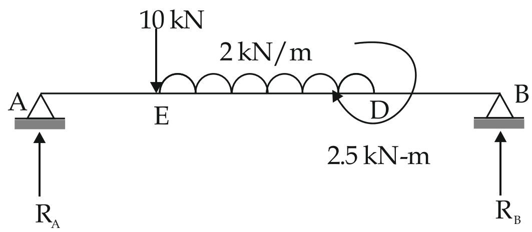
$R_B = 11.4375$ kN

$R_A = 14.5625$ KN

SF in AE [x from A]

$$S_x = R_A = 14.5625 \text{ KN}$$
$$S_A = S_E = 14.5625 \text{ KN}$$





SF in ED [x from E]

$$S_x = R_A - 10 - 2x$$
$$= 4.5625 - 2x$$
$$S_E = 4.5625 \text{ KN}$$
$$S_D = -11.4375 \text{ kN}$$
$$x = 2.28125 \text{ m}$$

For $S_x = 0$,

SF in DB [x from B]

$$-R_B = -11.4375$$
$$S_D = S_B = -11.4375 \text{ kN}$$

BM in AE [x from A]

$$M_x = R_A x = 14.5625\,x$$
$$M_A = 0$$
$$M_E = 116.5 \text{ kNm}$$

BM in ED [x from E]

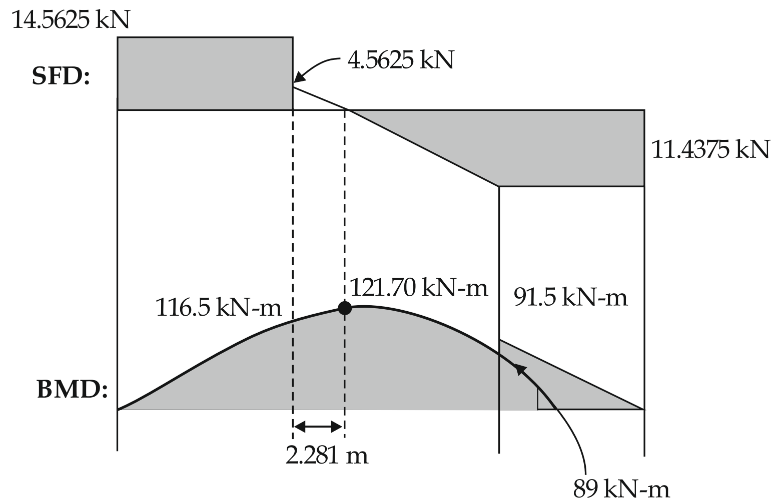
$$M_x = R_A(x+8) - 10x - \frac{2x^2}{2} = 116.5 + 4.5625x - x^2$$

$$M_E = 116.5 \text{ kNm}$$
$$M_D = 89 \text{ kNm}$$
$$M_{max} = 121.70 \text{ kNm}$$

BM in DB [x from B]

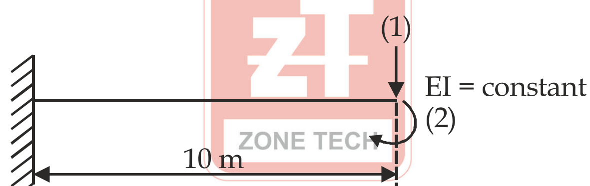
$$M_x = R_B x = 11.4375x$$
$$M_B = 0$$
$$M_D = 91.5 \text{ kNm}$$

Maximum bending moment = 121.7 kNm



Q.35

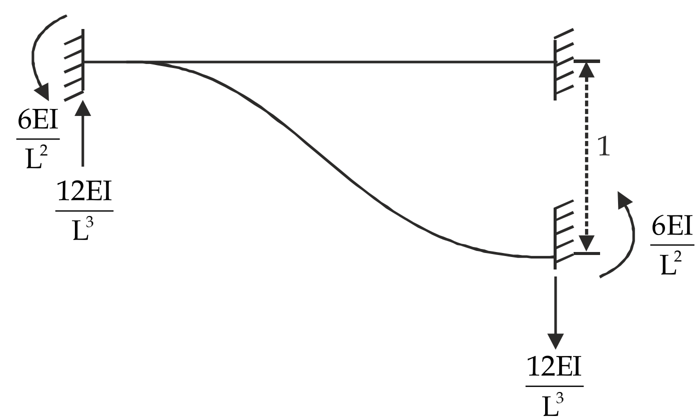
Sol. We have,



Stiffness matrix, $[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$

1st Column :

Applying unit displacement along (1)

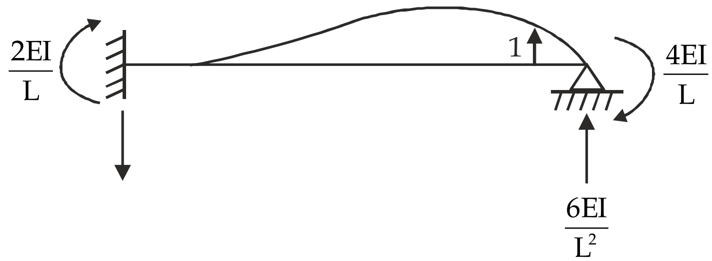


Hence, $K_{11} = \frac{12EI}{L^3} = \frac{12EI}{(10)^3} = 0.012EI$

$K_{21} = \frac{-6EI}{L^2} = -0.06EI$

2nd Column :

Applying unit displacement along (2)



$$K_{22} = \frac{4EI}{L} = 0.4EI$$

$$K_{12} = \frac{-6EI}{L^2} = -0.06EI$$

so, $$[K] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} = \begin{bmatrix} 0.012EI & -0.06EI \\ -0.06EI & 0.4EI \end{bmatrix}$$

Determination of flexibility matric:

Flexibility matrix [F] = K^{-1}

$\Rightarrow$ $$K^{-1} = \frac{\text{Adj } K}{|K|}$$

$$|K| = EI^2[0.012 \times 0.4 - (-0.06) \times (-0.06)]$$

$$= 0.0012\ EI^2$$

$$\text{adj } K = \begin{bmatrix} 0.4EI & 0.06EI \\ 0.06EI & 0.012EI \end{bmatrix}$$

$\therefore$ $$[F] = \frac{\text{adj}K}{|K|} = \frac{1}{0.0012EI^2}\begin{bmatrix} 0.4EI & 0.06EI \\ 0.06EI & 0.012EI \end{bmatrix}$$

$$= \begin{bmatrix} \frac{333.33}{EI} & \frac{50}{EI} \\ \frac{50}{EI} & \frac{10}{EI} \end{bmatrix}$$

Q.36

Sol. Given Data: B = 300 mm, D = 600 mm, A_{st} = 2060 mm², A_{sc} = 804 mm², d_c = 50 mm, d = D – d_c = 600 – 50 = 500 mm, M20/Fe 415

(i) Limiting depth of neutral axis:

$$x_{u,lim} = 0.48\ d = 0.48 \times 550 = 264\ mm$$

(ii) Actual depth of neutral axis:

Total compressive force = Total tensile force

$$0.36f_{ck}Bx_u + A_{sc}(f_{sc} - 0.45f_{ck}) = 0.87\ f_y\ A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 300 \times x_u + 804(f_{sc} - 0.45 \times 20) = 0.87 \times 415 \times 2060$$

$$\Rightarrow 2160x_u + 804(f_{sc} - 9) = 743763$$

$$\Rightarrow 2160x_u + 804f_{sc} - 7236 = 743763$$

$$\Rightarrow 2160x_u + 804f_{sc} = 750999$$

$$\Rightarrow x_u = \frac{750999 - 804f_{sc}}{2160}$$

Trial 1:

$$\text{Assuming } f_{sc} = 350\text{ MPa.} \quad (\text{for Fe 415})$$

We get, $$x_u = \frac{750999 - 804 \times 350}{2160} = 217.407\text{ mm}$$

$$\text{Value of } \varepsilon_{sc} = \frac{0.0035}{x_u}(x_u - d_c) = \frac{0.0035}{217.407}(217.407 - 50) = 0.002695$$

$$f_{sc}\text{ for } 0.002695 = 342 + \frac{(351 - 342)}{(0.00276 - 0.00241)}(0.002695 - 0.00241)$$

= 342 + 7.328-349.328 MPa

Trial 2:

$$x_u = \frac{750999 - 804 \times 349.328}{2160} = 217.657\text{ mm}$$

$$\text{Value of } \varepsilon_{sc} = \frac{0.0035}{217.657}(217.657 - 50) = 0.002695$$

Hence f_{sc} = 349.328 MPa and x_u = 217.657 mm (adopted)

$\because$ $x_u < x_{u,lim}$, hence section is under reinforced.

(iii) Moment of Resistance

$$MR = 0.36f_{ck}Bx_u \times (d - 0.42x_u) + (f_{sc} - 0.45f_{ck})A_{sc}(d - d_c)$$

= [0.36 × 20 × 300 × 217.657 × (550 – 0.42 × 217.657)]

+[(349.328 – 0.45 × 20) × 804 × (550 – 50)]

= 352.409 kN-m

Q.37

Sol. Given:

Pakoeco Factored load, P_u = 1100 kN

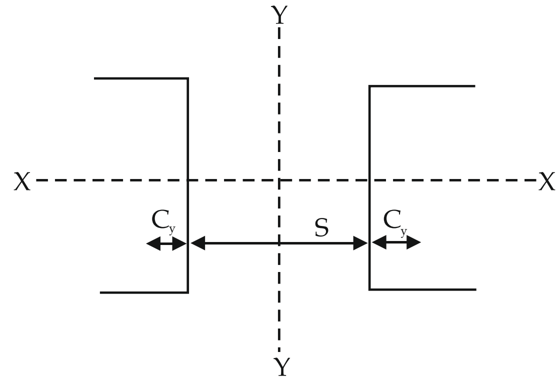
Unsupported length, L = 10 m

Since the column is restrained in position but not in direction at both ends.

Hence, Effective length, I = 1.0 L = 10 m

Calculation for spacing of channels :

2 ISMC 300 channels are spaced that the least radius of gyration of built up sections becomes as large as possible. To achieve this, channels are such spaced.



$$I_{YY} \geq I_{ZZ}$$

$$\therefore \quad 2\left[I_y + A\left(\frac{S}{2} + C_y\right)^2\right] \geq 2I_x$$

$$3.13 \times 10^6 + 4630\left(\frac{S}{2} + 23.5\right)^2 \geq 6.42 \times 10^7$$

$$S > 182.69 \text{ mm}$$

Provide 2 ISMC 300 channels at a spacing of 200 mm.

Check for compressive strength:

Minimum radius of gyration, $r_z = r_{min} = r_x = 118$ mm (Given)

Effective slenderness ratio, $\lambda_e = 1.05\frac{l}{r_z} = \frac{1.05 \times 10000}{118} = 89$

From the table 9(c)

Design compressive strength, $f_{cd} = 136 + \frac{136 - 121}{80 - 90}(89 - 80) = 122.5 \text{ N/mm}^2$

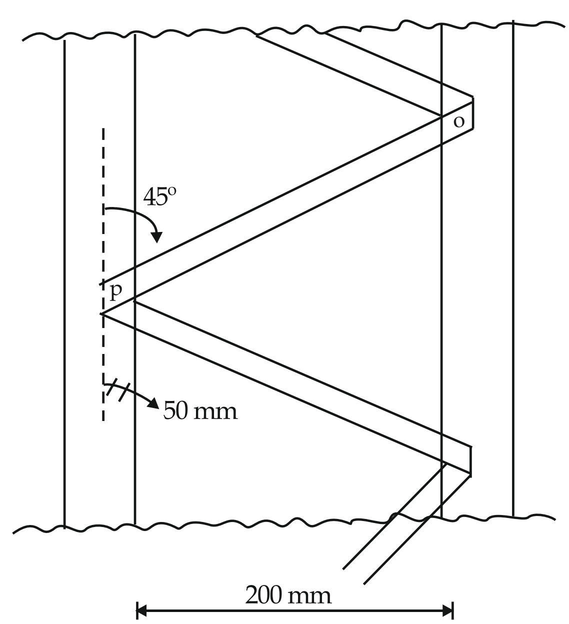
Design compressive strength, $P_d = f_{cd} \cdot A$

= 122.5 x (2 × 4630)

= 1134350 N

= 1134.35 kN > 1100 KN (OK)

Design of size of lacing rod



Maximum shear V_t = 2.5% of factored load

$$= \frac{2.5}{100} \times 1100 = 27.5 \text{ kN}$$

Compressive force in lacing bars $= \frac{V_t}{N} \operatorname{cosec}\theta = \frac{27.5}{2} \operatorname{cosec} 45^\circ = 19.45 \text{ kN}$

Length of flat lacing bar rod = (200 + 50 + 50)cosec45° = 424.3 mm

Effective length of flat lacing rod = 424.3 mm

Minimum thickness of lacing rod $= \frac{1}{40} \times 424.3 = 10.6 \text{ mm}$

Provide a flat section 50 ISF 12 mm

Minimum radius of gyration, r $= \frac{t}{\sqrt{12}} = \frac{12}{\sqrt{12}} = 3.46 \text{ mm}$

Slenderness ratio, $\lambda = \frac{l}{r} = \frac{424.3}{3.46} = 122.63 < 145$

From table 9(c)

Design compressive stress, $f_{cd} = 83.7 + \frac{83.7 - 74.3}{120 - 130}(122.63 - 120)$

Design compressive strength, $P_d = f_{cd} \cdot A$

= 81.23 × (50 × 12)

= 48736.7 N

= 48.74 kN > 19.45 kN

Hence lacing rod is safe against compressive force.

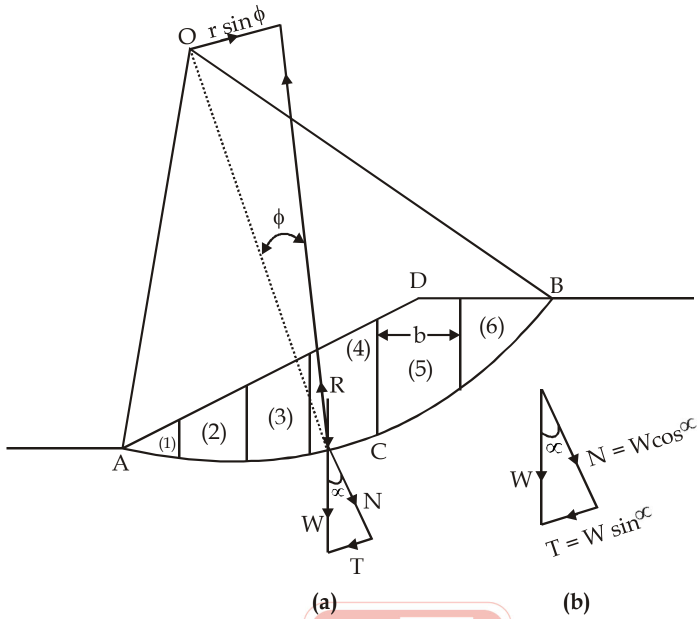
Q.38

Sol. Swedish Circle Method :

The actual shape of a slip surface in the case of finite slopes is curvilinear. For convenience, it is approximated as circular. The assumption of a circular. The assumption of a circular slip surface and its application for stability analysis of slopes was developed in Sweden. The method is known as the Swedish circle method or the method of slices.

Figure (a) shows a slope. Let AB be a circular surface with radius r and centre O. The trial failure wedge above the slip surfaace is divided into vertical slices by drawing vertical lines, as shown. The slices are usually of equal width, but not necessarily so. In case of non-homogenous slopes where the slip surface passes through more than one type of material, a vertical line is always located at the point where the slip surface passes from one material to the other.

Let us consider the equilibrium of one slice (say, No. 4). The slice is in equilibrium under the following forces.



(1) Weight (W) acting vertically through its centre of gravity.

(2) Cohesive force (C) acting along the curved surface in the direction opposite to the direction of probable movement of the wedge.

(3) Reaction (R) at the base inclined at angle ϕ to the normal, assuming the slippage is imminent.

(4) Reactions on the two vertical sides of the slice due to adjacent slices. However, in the Swedish circle method, it is assumed that the reactions on the two sides are equal and opposite and are, therefore, in equilibrium and do not affect the stability of the slice. Accordingly, only the first three forces are considered for the analysis.

The weight W is resolved into its normal component (N) and tangential component (T). Let us take the moments about the centre of rotation O of all the 3 forces.

Actuating or overturning moment,

$$M_0 = T \times r \qquad \text{...(a)}$$

The moment due to N-components is zero, as those components always pass through O.

Resisting moment,

$$M_R = (C\,\Delta\,L) \times r + R(r \sin\phi) \qquad \text{...(b)}$$

where ΔL is the length of the curved surface of the slice.

Resolving the forces is radial direction,

$$N = R \cos f \;\text{ or }\; R = N/\cos f \qquad \text{...(c)}$$

or $$R \sin f = N \tan f \qquad \text{...(d)}$$

From Eqs. (b) and (d),

$$M_R = (C\Delta L)r + N\,r\tan\phi \qquad \text{...(e)}$$

The factor of safety for the slice is equal to the ratio of the resisting moment (M_R) and the overturning moment (M_0). Thus

$$F_s = \frac{r[c\Delta L + N\tan\phi]}{Tr} = \frac{c\,\Delta\,L + N\,\tan\phi}{T}$$

The factor of safety of entire wedge is given by

$$F_s = \frac{\Sigma c\, \Delta L + \Sigma N \tan\phi}{\Sigma T} \quad \text{...(i)}$$

If c and ϕ are constant,

$$F_s = \frac{c L_a + \tan\phi \Sigma N}{\Sigma T} \quad \text{...(ii)}$$

where L_a = length of the entire slip surface $= \Sigma \Delta L$

The components N and T are determined by drawing force triangles as shown in Fig. (b). If the angle α which the normal makes with the vertical is measured, the components can be computed as under

$$N = W \cos a, \text{ and } T = W \sin a$$

The length ΔL of the arc is given by b sec α, where b is the width of the slice.

Therefore, Eq. (i) can be written as

$$F_s = \frac{\Sigma\, c\, b \sec\alpha + \Sigma (W \cos\alpha) \tan\phi}{\Sigma W \sin\alpha}$$

It may be noted that the tangential component T may be negative i.e. in the direction opposite to that of movement for some of the slices near the toe.

The procedure can be summarised as under :

- Take a trial wedge and divide it into 6 to 12 vertical slices.
- Determine the weight of each slice and its line of action.

  For convenience, the weight is generally taken proportional to the middle ordinate of the slice and it is assumed to have line of its action through the middle of the slice.
- The weight is resolved (analytically or graphically) into normal and tangetial components.
- The curved length ΔL of each slice is measured or computed.
- The factor of safety is determined from eq. (iii) or Eq. (i)

The calculations are generally done in a tabular form. The stability analysis is repeated for a number of trial surfaces. The circle which gives the minimum factor of safety is the most critical circle.